

Creating a Classroom Culture that Supports Productive Struggle: Pre-service Teachers'  
Reflections on Teaching Mathematics

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April 13, 2018

New York, NY

### Abstract

The purpose of this instrumental case study was to examine pre-service teachers' (PSTs) reflections on mathematics teaching and learning as it relates to the practice of supporting productive struggle. The research team created a framework aligning instructional features/teacher strategies that support productive struggle with related perspectives, organizing these by classroom culture/environment, treatment of content, and communication to support reasoning. The current analysis reports on PSTs perspectives related specifically to classroom culture/environment. Findings show the variant ways in which PSTs perspectives are both aligned and misaligned as well as considerations for leveraging these perspectives in mathematics teacher education.

*Keywords:* case study, mathematics education, pre-service teachers, productive struggle, teacher education

\*This study has been reviewed and approved by an institutional review board.

### Purpose

This paper seeks to address the 2018 AERA theme to confront “the struggles for public education” (AERA, 2018, p. 1) by investigating the beliefs and perspectives of elementary pre-service teachers (PSTs) and how they may be leveraged to support their development as effective teachers of mathematics. Notions of inclusivity, access, and equity are critical concerns in mathematics education as indicated by the National Council of Teachers of Mathematics (NCTM) and are considered one of the five “essential elements” of successful mathematics programs (2014, p. 59). According to NCTM, equity refers to practices that “promote access and attainment for all students” (NCTM, 2000, p. 12). Ellis (2008) and Ellis and Berry (2005) argue that an obstacle to access and equity is instruction that seldom focuses on meaningful learning but rather foregrounds rote skill and procedural learning. NCTM (2014) echoes this perspective, stating “we must change a range of troubling and unproductive realities,” including prioritizing “learning procedures without any connection to meaning, understanding, or the applications that require these procedures” (p.3).

These “unproductive realities” highlight what Flores (2007) describes as “opportunity gaps” (p. 40), where “differential instructional opportunities” are the main culprit for restricting learners’ access to engage in meaningful mathematics learning. NCTM advocates eight research-based practices that serve to support meaningful learning opportunities of all students. In our ongoing efforts to address many of the access and equity issues common in K-12 mathematics learning environments, we seek to focus PSTs’ development on these eight practices. Building on prior research concerning one particular practice, *support productive struggle in learning*, we examined PSTs’ reflections on teaching and learning to understand the various ways their own experiences might influence their perspectives concerning mathematics teaching.

### Theoretical Framework

The idea of struggle as an important aspect of learning has long been advocated by Dewey (1929; 1910/1997) and Pólya (1957), who noted the contribution of effort in developing new ideas and understandings. Cognitive learning theorists (Hatano, 1988; Piaget, 1960; Skemp, 1971; Steffe, 1991; von Glaserfeld, 1991) also argue for the significance of struggle in the learning process. Kapur (2008) describes the construct, *productive failure*, arguing that struggle not leading to success can still contribute to learning. Research results support the benefits of engaging in struggle while learning. Hiebert and Grouws (2007) concluded that providing opportunities for students to “struggle with important mathematics” (p. 387) plays a key role in learning that results in conceptual understanding. Yet, it is difficult for many teachers to engage their students in productive struggle while learning mathematics. Many (if not most) did not experience productive struggle when learning mathematics. In many cases, struggle in mathematics was/is perceived as a negative (Hiebert & Wearne, 2003) and, therefore, the teachers’ role is to prevent, avoid, and/or remove struggle from learning (Stein, Smith, Henningsen, & Silver, 2009; Stigler & Hiebert, 2004). Such cultural beliefs can be “obstacles to consistent implementation of effective teaching and learning in mathematics classrooms” (Handal, 2003; Philipp, 2007). Changing these beliefs requires, in part, a change in what it means to do mathematics. Hiebert and Grouws note: “By struggling with important mathematics we mean the opposite of simply being presented with information to be memorized or being asked only to practice what has been demonstrated” (2007, p. 388). Rather, in this view, mathematics makes sense and learners are capable of making sense of the mathematics. Yet many parents, and even educators, view the teaching of mathematics through a lens that is informed by the way

they were taught; further, many fear that moving away from familiar practices will harm rather than improve student learning (NCTM, 2014).

NCTM plays a leading role in providing guidance for a vision of effective mathematics teaching and learning. In 2014, NCTM published *Principles to Actions: Ensuring Mathematical Success for All*, which describes a framework for supporting all students in learning mathematics. Here, NCTM the eight research-informed teaching practices that represent “a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics” (p. 9). In 2017, the Association of Mathematics Teacher Educators (AMTE) released the *Standards for Preparing Teachers of Mathematics*, which “articulate a national and comprehensive vision for the initial preparation of teachers of mathematics in Pre-K–12” (p. x). This document emphasizes the need for well-prepared teachers to understand and implement effective teaching practices, “such as those described in *Principles to Actions*” (p. 15). One of the practices suggested by NCTM and AMTE is supporting productive struggle in learning mathematics. NCTM (2014) elaborates on this practice: “[e]ffective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships” (p. 10). While a practice-based perspective focuses on the work of effective mathematics teaching, there is a recognition that teacher education must also “attend to the knowledge and orientations that underlie effective teaching” (Ball & Forzani, 2011, p. 19). In the sections that follow, we share a framework for considering productive struggle emphasizing both instructional features and aligned perspectives.

### **Perspectives That Align with Productive Struggle**

In order to understand how to effectively support our PSTs' development of the practice of supporting productive struggle in mathematics, we first examined more carefully the complexity this practice. We began by examining the literature to delineate instructional features and aligned perspectives that support product struggle. Our efforts uncovered a diverse array of instructional features/teaching strategies that support the practice (Doerr, 2006; Engle, 2006; Franke et al., 2015; Gresfali, Martin, Hand, & Greeno, 2009; Hiebert & Grouws, 2007; M. Kapur & Bielaczyc, 2012; NCTM, 2014; Mary Kay Stein, Smith, Henningsen, & Silver, 2009; Warshauer, 2015a, 2015b). We categorized these into three broad areas: classroom culture/environment, treatment of mathematics content, and communication to support mathematical reasoning. With these in mind, we articulated perspectives that align with these features. Here, we define perspectives as "a particular way of viewing things that depends on one's experience and personality" ("Perspective Definition," 2017). It encompasses beliefs, perceived relationships, assumptions, and biases that may have resulted from one's experiences. Table 1 summarizes these instructional features/teacher strategies and related perspectives.

Table 1

*Instructional Features/Teacher Strategies that Support Productive Struggle and Related Perspectives*

<b>Instructional Feature/Teacher Strategy</b>	<b>Related Perspectives</b>
<i>Classroom Culture/Environment</i>	
<ul style="list-style-type: none"> <li>• Establish a supportive learning environment (M. Kapur &amp; Bielaczyc, 2012)</li> <li>• Provide access to tools that support thinking (NCTM, 2014)</li> <li>• Allow students time to struggle (NCTM, 2014; Warshauer, 2015a, 2015b)</li> <li>• Explicitly encourage perseverance in sense-making (Gresfali et al., 2009; NCTM, 2014)</li> <li>• Establish shared authority for creating and validating the mathematics (Doerr, 2006;</li> </ul>	<p><i>All students can learn</i></p> <ul style="list-style-type: none"> <li>• growth mindset/all students can improve understanding with effort</li> <li>• the teacher is responsible for supporting the learning of all students</li> <li>• a supporting, caring teacher is influential in creating a positive learning environment for students</li> </ul> <p><i>Nature of mathematics</i></p>

Instructional Feature/Teacher Strategy	Related Perspectives
<p>Franke et al., 2015; Gresfali et al., 2009; NCTM, 2014; Warshauer, 2015a, 2015b)</p> <ul style="list-style-type: none"> <li>Communicate expectations that confusion and mistakes are natural and opportunities for learning (NCTM, 2014)</li> <li>Acknowledge struggle as an important part of learning mathematics (Warshauer, 2015a, 2015b)</li> <li>Establish a community engaging all members in learning mathematics (Engle, 2006; Franke et al., 2015).</li> </ul>	<ul style="list-style-type: none"> <li>mathematics is more than just producing steps outlined by the teacher or the text</li> <li>authority lies in the mathematics; students can determine “correctness” by using the mathematics (not the teacher or text)</li> </ul> <p><i>Student agency</i></p> <ul style="list-style-type: none"> <li>learners are sense-makers in the mathematics/have agency</li> </ul> <p><i>Struggle is part of learning</i></p> <ul style="list-style-type: none"> <li>struggle is a beneficial feature of the learning process</li> <li>mistakes and errors are natural and can be opportunities for learning</li> </ul>
<i>Treatment of Content</i>	
<ul style="list-style-type: none"> <li>Implement tasks that promote reasoning and problem solving (NCTM, 2014; Mary Kay Stein et al., 2009; Warshauer, 2015a)</li> <li>Maintain cognitive demand of tasks during implementation (Mary Kay Stein et al., 2009)</li> <li>Anticipate and plan for students’ solution approaches, including areas of struggle (NCTM, 2014; Stein et al., 2009)</li> <li>Open up examination of mathematics to multiple solution approaches and ideas (NCTM, 2014; Stein et al., 2009)</li> </ul>	<p><i>Nature of mathematics</i></p> <ul style="list-style-type: none"> <li>mathematics is more than just producing steps outlined by the teacher or the text; mathematics is dynamic and connected</li> </ul> <p><i>Struggle is part of learning</i></p> <ul style="list-style-type: none"> <li>struggle is a beneficial feature of the learning process</li> <li>mistakes and errors are natural and can be opportunities for learning</li> </ul> <p><i>Student agency</i></p> <ul style="list-style-type: none"> <li>learners are sense-makers in the mathematics/have agency</li> </ul>
<i>Use Communication to Support Reasoning</i>	
<ul style="list-style-type: none"> <li>Pose questions to scaffold and support but not rescue (Engle, 2006; Franke et al., 2015; Hiebert &amp; Grouws, 2007; NCTM, 2014; Warshauer, 2015b).</li> <li>Listen to students’ ideas and pose questions to scaffold, support, elicit and build on their thinking (Doerr, 2006; Engle, 2006; NCTM, 2014; Warshauer, 2015a, 2015b)</li> <li>Facilitate discourse focused on explanation and justification (NCTM, 2014).</li> <li>Invite students to participate in mathematics and support their efforts (Franke et al., 2015).</li> </ul>	<p><i>Struggle is part of learning</i></p> <ul style="list-style-type: none"> <li>struggle is a beneficial feature of the learning process</li> <li>mistakes and errors are natural and discussing them creates opportunities for learning</li> </ul> <p><i>Student agency</i></p> <ul style="list-style-type: none"> <li>learners are sense-makers in the mathematics/have agency</li> </ul> <p><i>Nature of mathematics</i></p> <ul style="list-style-type: none"> <li>explanations require mathematical justifications (not just “what” but also “why”)</li> </ul>

Instructional Feature/Teacher Strategy	Related Perspectives
<ul style="list-style-type: none"> <li>Assign competence to students' mathematical ideas (Doerr, 2006; Engle, 2006; Franke et al., 2015).</li> </ul>	

Clearly, this practice is complex, requiring working with the myriad layers and considering the various manifestations in classrooms. The list of features and aligned perspectives suggested here are those highlighted in the literature as being included in instruction aimed at engaging students in struggle that leads to learning. It is not exhaustive or conclusive, and no one feature encompasses the practice. Rather, we argue, it is the combination of and interaction among these features that is most likely to support productive struggle. An elaboration of features and perspectives related to classroom culture, treatment of content, and use of communication follows.

### **Classroom Culture/Environment**

Kapur (2010, 2011) and Kapur and Bielaczyc (2012) emphasized the importance of creating a learning environment that values persistence. Teachers can do so by explicitly emphasizing expectations of students that focus on effort and flexibility in solution methods and strategies rather than prioritizing outcomes. NCTM (2014) and Gresfali et al. (2009) also argued for the importance of emphasizing perseverance in making sense of mathematics. To support students' perseverance, Warshauer's (2015a, 2015b) and Stein et al.'s (2009) work advocated providing time for students to struggle with mathematics and making explicit the role that struggle plays in learning. Other research (e.g., Doerr, 2006; Franke et al., 2015; Gresfali et al., 2009) highlighted the importance of establishing a mathematical learning community, making use of students' contributions to the work of the mathematical community, valuing students' thinking, and positioning students' ideas as valid mathematical considerations. We categorized



these and similar findings as instructional features related to classroom culture/environment.

Aligned perspectives fell under four categories: 1) all students can learn/growth mindset (teacher is responsible for supporting the learning of all students), 2) student agency (learners are sense-makers of the mathematics), 3) nature of mathematics (authority lies in the mathematics), and 4) struggle is part of learning (mistakes are natural occurrences in the learning process).

### **Treatment of Mathematics Content**

Another category of instructional features/teacher strategies that support productive struggle relates to the treatment of mathematics content. A feature in this category is the selection of rich tasks that engage students in thinking and reasoning and highlight the mathematics content to be learned (Kapur, 2008; NCTM, 2014; Stein et al., 2009; Warshauer, 2015a). The design of such tasks must be informed by a clear understanding of students' prior knowledge, specifically that which relates to the current learning goals (Manu Kapur, 2011). Further, implementation of the tasks must maintain high cognitive demand (Mary Kay Stein et al., 2009). Thus, teachers should "resist the impulse to provide assistance or help" as soon as students experience struggle (Manu Kapur, 2011, p. 575). Additionally, teachers should welcome multiple approaches and ideas to solutions, encouraging open discussion of the mathematical merits of each (M. Kapur & Bielaczyc, 2012; NCTM, 2014; Mary Kay Stein et al., 2009). To facilitate this, teachers must anticipate the multiple ways students may approach a solution and plan for areas of struggle, considering both how to support students without rescuing them and how to create opportunities to learn from students' developing understandings (NCTM, 2014; Mary Kay Stein et al., 2009; Warshauer, 2015a). Perspectives aligned with these instructional features include: 1) the nature of mathematics (mathematics is dynamic and connected), 2) the

role of struggle (struggle is a natural and important part of the learning process), and 3) student agency in mathematics.

### **Communication to Support Mathematical Reasoning**

The third category, communication to support mathematical reasoning, includes features such as: posing questions and comments that scaffold and support students' efforts (Engle, 2006; Franke et al., 2015; Hiebert & Grouws, 2007; NCTM, 2014; Warshauer, 2015b), eliciting, listening and responding to, and making use of students' ideas (Doerr, 2006; Engle, 2006; NCTM, 2014; Mary Kay Stein et al., 2009; Warshauer, 2015a, 2015b), facilitating discourse focused on explanation and justification (NCTM, 2014; Mary Kay Stein et al., 2009), inviting and supporting students' participation in mathematics (Franke et al., 2015), and assigning competence to students' mathematical ideas (Doerr, 2006; Engle, 2006; Franke et al., 2015). Features in this category address actions teachers may take to elicit students' thinking as well as how teachers may respond to students' contributions. For example, teachers' orientations toward listening to students' ideas can influence students' agency and authority in mathematics classrooms (Doerr, 2006). Teachers who listen in order to understand (rather than to evaluate) and use this understanding to support students' mathematical growth, promote a view of mathematics as something that is connected and logical and of students as those who can make sense of ideas. Further, teachers who not only invite students' ideas, but also support students as they develop and communicate their thinking, engage them in meaningful participation in the larger learning community (Franke et al., 2015). Perspectives aligned with these features include: 1) struggle is part of learning (examining errors creates learning opportunities), 2) students have mathematical agency (learners are sense-makers), and 3) the nature of mathematical explanations (explanations require mathematical justifications).

### **Research Methods, Data Sources, and Analysis**

We used instrumental case study (Stake, 2005), a method aimed at understanding a phenomenon within the context in which it occurs. Case study research allows researchers to focus on the complexities of a phenomenon within a single case and across multiple cases (Baxter & Jack, 2008; Stake, 2005). Stake defines the purpose of “instrumental cases” as those that “provide insight into an issue or redraw a generalization. The case is of secondary interest, it plays a supportive role, and it facilitates our understanding of something else” (Stake, 2005, p. 445). In this study, the “something else,” or phenomenon, was to understand the variant perspectives pre-service teachers bring to their elementary teacher education program and how these do or do not align with the practice of supporting productive struggle. Two research questions guided our investigation: *1) In what ways do teachers reflect on and communicate perceptions about teaching and learning mathematics, both in their learning stories and their reflections as future teachers? 2) How do these perceptions align with the mathematics teaching practice of supporting productive struggle?*

Data includes 30 mathematical learning stories and 7 interviews from elementary education majors towards the end of their program from 2014-2016. Pre-service teachers wrote the mathematical learning stories as an assignment in their mathematics methods course taught by the researchers and were asked the following: Write a mathematics autobiography focusing on key moments you have experienced as a student or as a teacher. Elaborate on what makes these moments stand out for you. Follow-up interviews were conducted after students completed the course with the purpose of elaborating on their learning experiences as well as to elicit their current perspectives about teaching and learning mathematics. Using holistic coding (Saldaña, 2013), we “chunked” all data sources. First, we read through all the interview transcripts and

learning stories, line by line, and chunked the data into collections and nested collections within Transana, a multi-user qualitative data analysis software program. These chunks usually consisted of a mathematics story spanning 2 sentences to 2 pages. We assigned temporary and tentative names to these collections to help us identify the various ways they experienced and perceived their own mathematical learning. We noticed that most of these stories conveyed either a positive learning experience, a negative learning experience, or a reflection/perception about their future teaching – sometimes these experiences and reflections/perceptions were connected in their stories. As we continued collaboratively analyzing these emergent patterns in the data, we compared their experiences, perspectives, and reflections to the instructional features and teaching strategies related to the practice of supporting productive struggle in learning mathematics (see Table 1).

### **Results and Discussion**

Data indicates that many PSTs' reflections are reactions to their own experiences as learners of mathematics as well as what they anticipate may be the experiences of their future students. In the sections that follow, we share examples related to the three features outlined in Table 1, classroom culture/environment, treatment of content, and use of communication, discussing the alignment/misalignment with practices and perspectives related to productive struggle. Because the overwhelming majority of our PSTs' reflections relate to classroom culture/environment, this comprises the main focus for the results discussed in this paper.

We begin by sharing George's description of his experiences in his favorite college mathematics class. George first talked about the difficulty of the class ("It was really hard! I will be the first to admit that I really struggled with it."), but also emphasized the satisfaction he felt when he was able to persevere. He described the support of his peers and his instructor. For

example, his instructor allowed students to think through the problems and didn't step in to "rescue" students ("She didn't just tell us the answer...she would let us struggle."). Further, his instructor acknowledged and assigned competence to his approaches to problems when they differed from what was discussed in class or in the text. The learning environment that George describes is one that exemplifies many of the features and strategies of classroom cultures that support students' engagement with productive struggle: allowing students time to struggle as well as establishing a supportive environment, shared authority, and a community of learning.

When George reflects on views of teaching and how he hopes to establish his own classroom culture, he connects to many of the features described in his experience:

Just not letting my students give up, I feel like is best...challenging them enough to where they can do it; to where they will struggle, but they can finish through. Or they can work together to finish through...Just knowing my students...how they learn best...I know it's okay to struggle and you probably should struggle to learn effectively.

He continues:

I think, letting students struggle a little bit is a good thing. Which is kind of what I've learned a little in those classes. Guiding them through rather than showing them how to do it, sometimes, is better.

George's reflections touch on many instructional features and related perspectives of a classroom culture that would support productive struggle in learning mathematics. Most importantly, George expresses a perspective that struggle is not just part of learning but *necessary for effective learning*. He seeks to establish a supportive environment (Kapur & Bielaczyc, 2012) by getting to know his students and understanding how they learn best. In addition, he connects to perspectives of recognizing and valuing the diversity of skills and understandings that are present

and using that to support learning for all students (National Research Council, 2005). He talks about allowing for and even encouraging struggle, ensuring that the struggle is centered on mathematics that is within reach of the learner (Hiebert et al., 1997; Hiebert & Grouws, 2007).

Although George touches on these ideas, he is not always specific in describing particular actions he might take; rather, he speaks in generalities. Even when he talks about allowing students to struggle, he is hesitant: he discusses letting students “struggle a little bit” and notes that not telling them “sometimes is better.” Although he mentions several aspects related to productive struggle, he does not touch on the wide range of instructional features listed in Table 1. This was true of many of our PSTs. Many note a desire to create a positive and/or open learning environment, citing things like helping students to not be afraid to ask questions, being patient, and being approachable. Others discuss explicitly encouraging students to persevere, allowing time to struggle, and encouraging a growth mindset. However, they do not describe specific ways in which they may accomplish these goals.

In fact, for most PSTs, the desire to make mathematics class a positive experience for students took precedence over anything else. For example, Natalie discusses how she sees herself and her classroom as a future teacher of mathematics:

I want to make math fun and exciting for all of my students. Most importantly I want my students to know that everyone struggles with certain concepts in school but that we each can get better with help and through practice. Math to me is a journey, I will never stop learning mathematics as long as I am a teacher. To me, the answer will never change but the approaches to get there are endless. I am excited to learn from my students' thinking and see where their minds take me throughout their mathematical journey.

Like George, Natalie wishes to create a supportive learning environment where students understand that struggle is something that “everyone” experiences. She talks about improving through perseverance. Further, she suggests that she can and will learn from her students, positioning them as mathematical sense-makers and sharing authority for developing mathematical ideas. Natalie begins by emphasizing that she wants mathematics to be “fun and exciting.” This was common. As PSTs expressed general wishes for creating a positive environment for their students, they often focused on making mathematics fun and enjoyable.

Alicia reflects on her future classroom:

I want all my students to be successful in math while enjoying. However, I think my first goal is getting my students to look forward to learning math rather than dreading math class. I want to find ways to interest students in math such as relating it to real world or even whatever they are interested in. By doing this I think students will be more likely to enjoy and look forward to our math class.

Alicia wishes to support her students in developing a positive disposition towards mathematics by creating a positive learning environment. She wants to tap into their interests in the hope that they will then enjoy math class. In these examples, PSTs clearly wish to create a positive experience for their students. Often, this was in response to their own negative experiences when they dreaded mathematics. While some instructional features and perspectives related to the practice of supporting productive struggle in learning are evident in these reflections (e.g., establishing a supportive learning environment, encouraging perseverance, a supporting, caring teacher), the main focus of these seem to be on having “fun” but not specifically in a way that connects to significant learning.

While these examples of PSTs' reflections align with several features of classroom environments that support productive struggle, other PSTs' reflections reveal ideas that would be counter-productive to engaging in this practice. For example, Molly reflects on her own experiences as a learner of mathematics and how those influence her vision of her future classroom.

I think that because of my own personal struggles in math, I will take much more time in understanding my students' individual needs and never give up on believing in my students' abilities to be successful. I hope to be the teacher who sees a student struggling and takes the extra time to work with them after school and help them in the areas they need. This will not only require help and cooperation from me, but it will also require help and cooperation from parents.

Molly sympathizes with her future students who may struggle in mathematics. Whereas her reflection seems to start out with a productive view (taking time to understand students' individual needs and having high expectations of her students), her reflections on teaching focus on identifying struggle and supporting this with extra work time, time after school, and the importance of families being involved in this process. This view seems to suggest that she views struggle not as a natural part of the learning process but as an exception, something that is out of place and needs to be "fixed" outside of regular instructional time. Thus, there is little reflection about what would change in her primary math teaching practice aside from sympathy and taking time to identify her students' needs.

Anne talks about her experience substitute teaching in a special education class:

I had to teach some math and 4 boys were stuck on a problem. I was like, "oh goodness they are never going to get it!" And one day I was like, "okay this is how I got it, just



think of it this way,” and I showed them what I thought. They all got it and the aid was impressed and I was like, “Yeah, I could do this – even with special ed. kids.” I can get them to get it...I realized that I could do it and I just made it fun. I was like, “this is how I think of it” and if I get it and this is how I think of it, then maybe it will help them get it.

Anne expresses satisfaction that she was able, in her view, to help the students achieve something. Her desire to help students, indicated here and in other reflective comments, expresses her wish to be caring and supportive. However, what appears to be absent is a perspective that all students can learn and are mathematical sense-makers. This leads her to remove struggle, positioning her as the sense-maker and the students as passive. Not only does this remove agency from the students, struggle is not seen as a natural and beneficial part of learning. Rather, it is something to be avoided (Hiebert & Wearne, 2003) and removed (Stein et al., 2009; Stigler & Hiebert, 2004). Of note is her comment, “even with special ed. kids.” Often sympathy is manifested as a byproduct of low expectations for students. These implicit biases of race, gender, and income influence expectations of students’ ability, how successes and failures are attributed, and how teachers respond to students (Cooper & Tom, 1984; Fennema et al., 1990; Rojas & Liou, 2017). Unfortunately, this often happens in ways that lessen rigor and demand, creating the instructional opportunity gaps that underlie inequitable classrooms.

### **Mathematics Content**

While reflections related to the treatment of mathematics content were less prevalent, there were a few commonalities across pre-service teachers’ reflections. For the most part, pre-service teachers viewed mathematics in a linear progression, with concepts building on previous learned concepts. They talk most about wanting to build a strong foundation and supporting learners to “understand each step so that the next one makes sense.” Lynn even communicates

that she wants to teach lower grades (K-2) to focus on teaching foundations and that teaching 3-5 is too late for this work. She writes that her own struggles learning mathematics throughout K-12 leave her “fearful...I'm going to be so overly cautious that I'm going to have a hard time moving on in the class.” She connects these ideas about focusing on foundations, her desire to teach lower grades, and how she will respond to struggle:

I don't need to cover as much in a year as you know let's say 4th grade or 5th grade. You know kindergarten and 1st graders don't cover as much. I also know that if I find a student like let's say that I can see they are able to do problems and repeat it with different numbers but they can't tell me why they are doing it or if you know I changed the question a little but they can't do it, they don't understand it. I'm not going to move them on from that. You are going to have – let's work on this again. Let's start from scratch again and build up to this because clearly, they don't understand it. When as I don't feel a teacher ever once did that with me.

While Lynn is considering the nature of understanding (e.g., transfer to problems worded differently), she still brings a perspective that treats mathematics as one of coverage and step-like (e.g., “start from scratch again”). For other pre-service teachers, there is a desire to make mathematics easy to understand. Some discuss a desire to use review strategies and drill of math facts that they found helpful in their own learning. Worrying to us is that there was almost no reflection from pre-service teachers regarding student agency; rather, the agency of the teacher was prioritized. And while they recognized that struggle is a natural part of the learning process, they didn't view this as important to learning but as something to fix. In addition, mathematics was never discussed as dynamic and connected, but as having a foundation in facts and procedures that need to be mastered before “moving on.”

In some cases, pre-service teachers did reflect on the use of various strategies students might use to solve problems, ones they learned in their mathematics education courses. For example, Karen writes:

It wasn't until this past Spring 2015 semester when I started re-learning new ways to approach mathematics. I really enjoyed the new strategies to help students understand, such as build a ten, double facts, and counting strategies. It helped me realize that there are entirely different ways I could have been taught the material. Now I know that there are multiple ways a particular student might approach a problem, and I have an understanding of what they might be seeing and doing to complete the problem. I can help them if they are struggling and I will not focus my teaching solely on learning a particular method, but instead on the way the student best learns.

While we see this reflection as a positive treatment of mathematics content, there is still little that explicitly considers the ways that she might elicit and connect student strategies and thinking. Mary's reflection is similar and seems to indicate the value in sharing various approaches to solving problems:

There are so many more different methods to doing it. And maybe that's what I meant by pedagogy, just different methods and strategies. And also I guess different ways rather than them just staying in their seats and just working on a worksheet like the share strategies I thought was really cool...that's something I would do. I guess there's different ways to teach math rather than the way I learned it. You know staying in your seat, worksheets, normal run of the mill type of stuff.

Mary reflects on her noticing of variant strategies used by students in the classrooms she has observed. She states that she would teach mathematics differently than she was taught (e.g.,

staying in seat, worksheets) and seems to indicate value in having students share strategies.

While this level of detail was atypical across the data, we still worry about the vagueness of strategy sharing as juxtaposed to her traditional experiences learning mathematics.

### **Communication**

Reflections on ways that pre-service teachers would use communication to support mathematical reasoning were mostly absent from the data. However, there was one pre-service teacher who recalled learning about orchestrating mathematical discussions in her mathematics education course. Lynn wrote:

We looked at math discussions and how those really help those who may be struggling to understand concepts. I like how students are able to share their strategies with the others.

If I would have had a math teacher who often conducted discussions, it would have made a world of a difference for my disposition towards math. I would have seen different ways of solving a problem, not just the way the teacher was showing us.

While Lynn is able to see the ways in which students communicating their strategies might benefit other students, it seems that the benefit is more to expose students to alternative strategies they could use rather than supporting them to connect and evaluate the benefits across these.

Because this excerpt was an anomaly in the data, we felt this was an area in which pre-service teachers might need more support in their teacher education program.

### **Significance**

These beginning reflections on teaching and learning can be leveraged to help PSTs better conceptualize and practice what it means to support struggle, a complex and multi-faceted practice. The list of features and aligned perspectives we have suggested is not exhaustive or conclusive and we plan to continue revising. Further, we do not perceive this list as a menu from

which only specific features can be selected and implemented to successfully engage in the practice; rather, it is the combination of these features that is most likely to support productive struggle. Thus, it will be important for teacher educators to support PSTs to unpack this multi-dimensional practice and examine its complexity. This would involve eliciting experiences and perspectives from PSTs as well as helping them reflect on areas of overlap with the productive struggle practice. Rather than approach PSTs' learning from a deficit perspective, we believe it is more beneficial to seek out and build on these overlaps.

As shown in the data, PSTs need opportunities to consider the wide range of features that support productive struggle and the ways in which these interact to support the mathematics learning environment. Even when PSTs express positive perspectives regarding a few of the features related to productive struggle, they may not necessarily consider approaches that support it or may, in fact, be counter to it. Many PSTs talk about being caring and supportive but do not necessarily discuss features such as mistakes and errors being opportunities to learn. Rather, their focus is on helping students feel good about themselves and the mathematics by making it fun and being a patient teacher.

We worry that these perspectives could cause these teachers to remove struggle, or at least focus efforts in ill-informed ways, particularly for students in groups for whom teachers' implicit biases may result in lower expectations and differential instructional opportunities. While motivation for these efforts may come from a positive place—desire to support students—Rojas and Liou (2017) point out that “[s]ympathy derived from perceived disadvantages can undermine students’ intellectual capacity” (p. 28). Rather, we want to support PSTs to expand their notion of caring as that which is grounded in high expectations for learners and the important role that struggle can play in creating agency and opportunity for learners.

Future efforts in our own work seeks to focus on communication to support mathematical reasoning. This was an area that only 3 of the 30 participants considered in their reflections. It is understandable perhaps that pre-service teachers are more focused on the establishment of their classroom environment, however, we view the discourse of the environment as a core way in which productive struggle emerges in this environment. There was a lack of consideration for actions teachers may take to elicit students' thinking as well as how they might respond to students' contributions. In current work redesigning our mathematics pedagogy courses, we are prioritizing this discursive practice, connecting this both to notions of productive struggle and as a practice that supports access and equity in teaching and learning. Part of this design includes incorporating videos and field experiences where pre-service teachers both focus on student thinking and ways of eliciting student strategies, mathematical justifications, and better understanding the learning opportunities created in these moments.

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